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ION LOSS BY COLLISIONAL UPSCATTERING

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1. INTRODUCTION

In at least one idealized version of the Polywell™/SCIF (Spherically Convergent Ion Focus) fusion scheme, the ions are injected at a point x_0 such that the magnetic field is sufficiently reduced from its maximum value that the ion orbits are nearly straight lines as they converge to the center of the device. Typically, this location will have a B-field in the range of 100 Gauss, compared to a B_{max} of several kG. This convergence in a strong potential ($e\phi \gg MV_{0i}^2$) produces a highly non-Maxwellian ion distribution, with a small dense core of nearly monoenergetic ions surrounded by a large mantle of less dense plasma made up of ions whose velocity is primarily radial.

There are two very distinct effects that collisions have on ion confinement. Firstly, as the ions pass through the geometric center of the device, collisional processes will alter their velocity distribution from nearly monoenergetic to Maxwellian. The lower end of this Maxwellian will continue to follow nearly straight line orbits, but the higher velocity ions, making a larger radial excursion into region of higher magnetic field, will be deflected by the B-field and will no longer converge to the center. In that sense, they are "lost" to the dense core which produces the bulk of the fusion reactions in the Polywell™/SCIF device.

There is a second collisional effect. As the ions traverse the mantle of lower density plasma outside the core, collisions can transfer momentum from primarily radial flow, which produces good spherical convergence, to a more isotropic distribution of velocities. This means an increase in local azimuthal velocity for the ions outside the core, which decreases convergence. The

decrease in convergence is a consequence of conservation of angular momentum, $rv_{\perp}$ = constant and energy $v_{\perp}^2 + v_r^2 - 2e\phi/M$ = constant, which limits the minimum radius an ion can access. This effect will be treated in a subsequent report.

The calculation[1,2] of velocity space diffusion due to collisions is straightforward; the standard results from the literature has been used to estimate the rates at which collisional processes in the dense core and external mantle produce diffusion of the ions in velocity[3-5]. The purpose of this note is to translate those calculations into estimates of the loss rate of fuel ions due to collisional processes.

2. FORMULATION OF THE EFFECTIVE LOSS RATE DUE TO UPSCATTER

Previous calculations[1] have shown that the change in ion velocity in a single pass through the dense core is small. Therefore the upscattering should be treated as a diffusion process (in velocity space) rather than a single scattering process. In analogy with the spatial diffusion problem, we use a continuity equation in velocity as the basis for the loss process

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial v^2}$$

where $D = (\delta v)^2/\tau$, with δv being the change in velocity produced in a time τ due to scattering in the dense core, and n the time dependent density of ions in velocity space.

Density in velocity space is the density of interest to ion confinement in the Polywell™/SCIF device, since it determines the ion convergence and thus the ion density in the core. The notion of ion loss in this device is unique among fusion schemes; loss means a decrease in ion core density resulting from loss in spherical convergence. Loss of ions from the device is not a significant energy drain, since source ions have very low energy. In fact, if an ion fails to converge to the core, it is better off lost from the system, since nonconvergent ions which linger in the system neutralize electrons, reducing the potential well.

3. CALCULATION OF THE DIFFUSION COEFFICIENT

The diffusion tensor in velocity space used in the Fokker-Planck equation is given by

$$D_{ij} = \frac{1}{2\tau} \langle \Delta v_i \Delta v_j \rangle \quad , \tag{1}$$

where Δv is the change in the velocity of a test particle in a time τ due to collisions with field particles. For an isotropic distribution of field particles, the diffusion tensor becomes diagonal. It can be expressed in terms of components parallel, $D_{\parallel}$, and perpendicular, $D_{\perp}$, to the direction of the initial velocity (before the collision) of the test particle. For reference, the derivation of the components of the diffusion tensor when the background field particles can be described by an isotropic Maxwellian is given in Appendix A, following Reference 1. In the case that the field particles are ions, denoted by a subscript 2, with a temperature T_2 defined by $(3/2)T_2 = m_i v_2^2/2$, the components of the diffusion tensor are given by

$$D_{\parallel} = \frac{e_1^2 e_2^2 n_2 \ln\Lambda}{4\pi\epsilon_0^2 m_i v_1} \; \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2} \quad , \tag{2a}$$

$$D_{\perp} = \frac{e_1^2 e_2^2 n_2 \ln\Lambda}{8\pi\epsilon_0^2 m_i^2 v_1} \left[\Phi\,(v_1/v_2) - \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2} \right] \quad . \tag{2b}$$

Here Φ is the error function, defined by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\zeta^2)d\zeta \quad , \tag{3a}$$

and $\Phi_1(x)$ is defined by

$$\Phi_1(x) = \Phi(x) - \frac{2x}{\sqrt{\pi}} \exp(-x^2) = \Phi(x) - x \frac{d\Phi}{dx} \quad . \tag{3b}$$

Figure 1 shows a plot of $\Phi_1(x)/2x^2$ and $\Phi(x) - \Phi_1(x)/2x^2$ (Ref. 2). In the limit that $x \to \infty$,

$$\Phi_1(x) \to 1 \quad ,$$

$$\Phi(x) \to 1 \quad .$$

For $x = 1$,

$$\Phi_1(x)/2x^2 = 0.214 \quad ,$$

$$\Phi(x) - \Phi_1(x)/2x^2 = 0.629 \quad .$$

As a first approximation, we assume that the dense ions in the core can be described by a Maxwellian distribution with a temperature T_2 corresponding to the radial energy of these energetic ions. We expect that this estimate for

the diffusion coefficient would be accurate to at least about a factor of 2 compared with other relevant field particle distributions, such as delta functions in velocity. For example, the diffusion coefficient for a test particle interacting with an isotropic distribution of field particles, each having the same magnitude of velocity, was derived in Reference 6. In a coordinate system in which one axis is parallel to the initial velocity vector v of the test particle, the diffusion tensor D_{ij} is diagonal, and is given by

$$\bar{D}_{\parallel} = \begin{cases} \dfrac{e_1^2 e_2^2 n_2 \ln\Lambda v_2^2}{4\pi\epsilon_o^2 3 m_i^2 v_1^3} & v_1 > v_2 \\[2ex] \dfrac{e_1^2 e_2^2 n_2 \ln\Lambda}{4\pi\epsilon_o^2 3 m_i^2 v_2} & v_1 < v_2 \end{cases} \tag{4a}$$

$$\bar{D}_{\perp} = \begin{cases} \dfrac{e_1^2 e_2^2 n_2 \ln\Lambda}{8\pi\epsilon_o^2 m_i^2 v_1}\left(1 - \dfrac{v_2^2}{3v_1^2}\right) & v_1 > v_2 \\[2ex] \dfrac{e_1^2 e_2^2 n_2 \ln\Lambda}{4\pi\epsilon_o^2 3 m_i^2 v_2} & v_1 < v_2 \end{cases} \tag{4b}$$

where again the subscripts 1,2 refer to the test, background particles, respectively. Comparing the expressions for $D_{\parallel}$, $D_{\perp}$ given in Eq. (2) for a Maxwellian distribution of field particles with the expressions for $\bar{D}_{\parallel}$, $\bar{D}_{\perp}$ given

in Eq. (4) for a delta function distribution of field particles, we obtain the following estimates. For $v_1 >> v_2$, we have approximately that $\tilde{D}\ /D\ \rightarrow 2/3$ and $\tilde{D}_\perp/D_\perp \rightarrow 1$. For $v_1 \sim v_2$, we have approximately that $\tilde{D}\ /D\ \sim 1.6$ and $\tilde{D}_\perp/D_\perp \sim 1.1$.

We note that the use of Eq. (4) might be a better approximation for experimental conditions in which the core density is in the lower density range of 10^{12}-10^{14} cm^{-3}, while the use of Eq. (2) might better apply to the high density case, $n \sim 10^{16}$-10^{18} cm^{-3}.

3.1 PARALLEL DIFFUSION DUE TO COLLISIONS IN THE CORE (UPSCATTER)

In this section we consider the effects of velocity upscatter due to scattering in the core. Upscatter in the radial direction due to collisions in the core would affect the ion loss because an upscattered ion could make a larger radial excursion into regions of larger B field, where it could be lost to the device in the sense described above. We use the parallel diffusion component D to describe the diffusion in radial velocity due to collisions in the core.

The dense core of the Polywell™/SCIF device occupies only a small fraction of the total volume of the machine. Since we are only interested in parallel diffusion due to collisions in the core, we multiply expression (2a) by the fractional time that a test ion spends in the core per pass through the device. It is assumed that the core has radius r_c and uniform density n_2. The transit time through the core is given approximately by $t_c = 2r_c/v_1$, where v_1 is the velocity of the test ion in the core region. The time for a test ion to transit through the entire device, however, is given approximately by $t_D = 2x_o/\bar{v}_1$, where x_o is the radial location at which the test ion was born, and $\bar{v}_1$ is

some average velocity of the test ion in the device. Thus the effective parallel diffusion coefficient due to collisions in the core is assumed to be given by

$$D_{\parallel c} = D_{\parallel} \left(\frac{t_c}{t_D}\right) = D_{\parallel} \left(\frac{r_c}{x_0}\right)\left(\frac{\bar{v}_1}{v_1}\right) \qquad . \tag{5}$$

We rewrite Eq. (5) in terms of the ion-ion collision time in the core, defined by

$$\tau_{iic} = \frac{\sqrt{3}\ 6\pi\epsilon_0^2 m_i^{1/2} T_2^{3/2}}{e_1^2 e_2^2 n_2 \ln\Lambda} \qquad , \tag{6}$$

(Ref. 1). Using this expression, we have that

$$D_{\parallel c} = \frac{1}{4} \left[\Phi_1\left(\frac{v_1}{v_2}\right)\right] \left(\frac{v_2}{v_1}\right)^3 v_2^2 \left(\frac{r_c}{x_0}\right)\left(\frac{\bar{v}_1}{v_1}\right) \frac{1}{\tau_{iic}} \qquad . \tag{7}$$

We will use the above expression to approximate the parallel diffusion coefficient for test ions resulting from collisions in the core of the device.

We need to write the diffusion coefficient in terms of system parameters. To do this, we use the results of Ref. 7 for the motion of an ion in a slab model of the Polywell[tm]/SCIF device. From that paper, the field particle ion velocity is given by

$$v_x^2 = v_{xo}^2 - 2e\,[\phi - \phi(x_o)]/m_i - [\omega_{ci}^2/(m+1)^2 R^{2m}][x^{m+1} - x_o^{m+1}]^2 \, , \tag{8}$$

where x denotes the radial axis situated midway between the magnetic cusps, x_o is the radial position at which the ions are born with a velocity v_{xo}, $\phi(x)$ is the electrostatic potential which is assumed to decrease as x decreases, R is the radius of the device, $\omega_{ci} = eB_o/m_i$, and the magnetic field varies as $B = B_o(x/R)^m$, where B_o is the magnetic field at R. In order to proceed with first approximation estimates, we assume that most of the energy of the core ions arises from their acceleration by the radial electrostatic field, with the term $2e\phi(x_o)/m_i$ being the dominant term for v_x^2 in the core. (It is assumed that $2e\phi(x_o)/m_i >> \omega_{ci}^2 x_o^{2(m+1)}/(m+1)^2 R^{2m}$ which is the condition for ion reflection to be avoided at the location where the ions are born, from Ref. 7.) Thus we approximate

$$v_2^2 \simeq \frac{2e\phi(x_o)}{m_i} \quad . \tag{9}$$

Since the test particle is actually one of the core particles, we assume that $v_1 = v_2(1 + f)$, where $f = \Delta v_1/v_2$, and Δv_1 is the upscatter in the test particle velocity due to collisions in the core. Then

$$D_{||c} \simeq \frac{1}{4}\,[\Phi_1(1 + f)]\,\frac{1}{(1 + f)^3}\,10^{16}\,(\frac{cm}{s})^2\,\phi\ (10\ kV)$$

$$\cdot\ (\frac{r_c}{x_o})(\frac{\bar{v}_1}{v_1})\ \frac{n_2(10^{12}cm^{-3})}{\phi^{3/2}\ (10\ kV)} \quad , \tag{10}$$

for $m_i = 2\, m_p$ for deuterium. The loss rate due to upscatter would then be given by

$$\tau_{loss} = \frac{v_2^2 f^2}{D_{\parallel c}}$$

$$\approx 4f^2(1 + f)^3 \frac{1}{[\Phi_1(1 + f)]} (\frac{x_o}{r_c})(\frac{v_1}{\bar{v}_1}) \frac{\phi^{3/2}\ (10\ \mathrm{kV})}{n_2(10^{12}\ \mathrm{cm}^{-3})}\ s\ . \qquad (11)$$

We estimate the value of f which could lead to ion "loss," in the sense of a broadened r_c, due to increased deflection by the higher B-field as the upscattered ion makes a larger radial excursion. To do this, we estimate the new radial location x_o' at which the upscattered ion's radial energy decreases to its birth energy. Neglecting scattering, we assume the ion radial energy profile is

$$E_r = E_o + e\phi_{max} \left[(\frac{x_o}{R})^p - (\frac{x}{R})^p \right] \quad , \qquad (12)$$

where typically p is related to the magnetic-field profile by $p \cong m$. Due to upscatter in the core, the ion receives a kick in energy ΔE_r,

$$\Delta E_r - e\phi_{max} (\frac{x_o}{R})^p\ 2f \quad . \qquad (13)$$

Then $E_r + \Delta E_r = E_o$ when

$$\frac{x_o'}{R} = (1 + 2f)^{1/p} \left(\frac{x_o}{R}\right) \quad . \tag{14}$$

The perpendicular deflection of the ion by the magnetic field, ΔV, at the ion birth location is related to the core radius by[7]

$$\frac{r_c}{R} = \frac{\Delta V}{V} \quad . \tag{15}$$

The increase in core radius, Δr_c, is then related to the change in ΔV, $\Delta V(x_o') - \Delta V(x_o)$, by

$$\frac{\Delta r_c}{r_c} = \frac{\Delta V(x_o') - \Delta V(x_o)}{\Delta V(x_o)} \quad . \tag{16}$$

The perpendicular deflection of the ion by the magnetic field was estimated in Ref. 7, viz:

$$\frac{\Delta V(x_o)}{V} \approx \frac{\omega_{ci}^2}{(m + 1)^2} \left(\frac{x_o}{R}\right)^{2m} \frac{x_o^2}{[2e\phi(x_o)/m_i]} \quad . \tag{17}$$

At x_o',

$$\frac{\Delta V(x_o')}{V} \approx \frac{\omega_{ci}^2}{(m + 1)^2} \frac{1}{R^{2m}} \frac{(1 + 2f)^{(2m+2/p)} x_o^{2m+2}}{[2e\phi(x_o')/m_i]} \quad , \tag{18}$$

so that the change in ΔV due to a larger radial excursion to x_o' is ($f \ll 1$)

$$\frac{\Delta V(x_0') - \Delta V(x_0)}{\Delta V(x_0)} \approx \frac{2(2m + 2)}{p} f \quad . \tag{19}$$

Thus

$$\frac{\Delta r_c}{r_c} \approx \frac{2(2m + 2)f}{p} \quad . \tag{20}$$

We assume the ion "loss" rate to be given by Eq. (11) evaluated with f such that $(\Delta r_c/r_c) \approx 2$. For $p = m = 3$, this gives $f \approx 1/2$. We make some estimates for τ_{loss}.

For $f = 1/2$, $x_0/r_c \sim 100$, and $\bar{v}_1 \sim v_1$, and $\phi = 10$ kV, $n_2 = 10^{12}$ cm^{-3},

$$\tau_{loss} \sim 600 \text{ s} \quad .$$

For reactor-grade parameters, with $f = 1/2$, $x_0/r_c \sim 100$, and $\bar{v}_1 \sim v_1$, and $\phi = 100$ kV, $n_2 = 10^{18}$ cm^{-3},

$$\tau_{loss} \sim 20 \text{ ms} \quad .$$

[illegible] $\bar{v}_1$ actually is $< v_1$ by factors of 2-6 in all real systems. Take $\bar{v}_1 \sim$ [illegible] v_1 then $\tau_{[illegible]}$ = [illegible]

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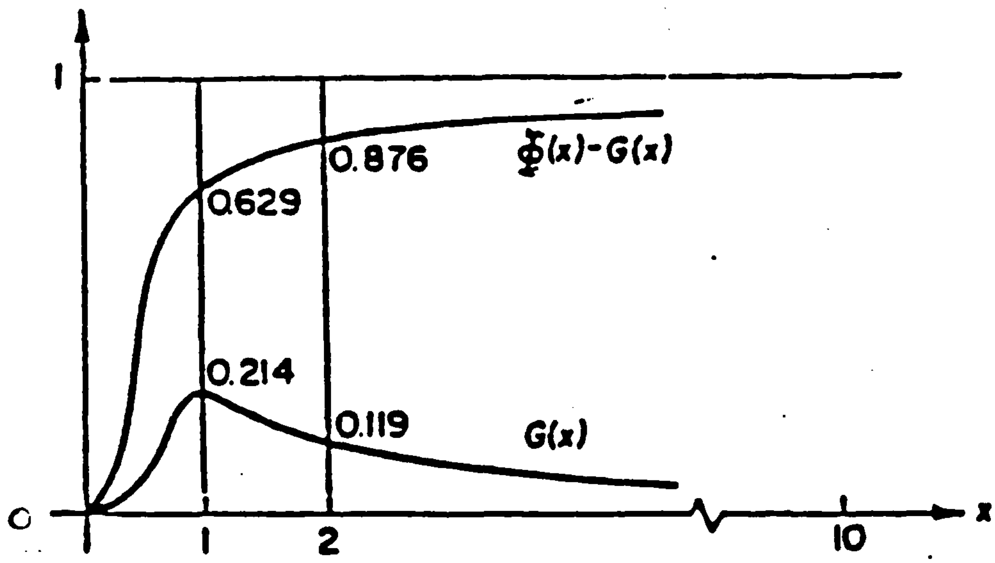

G(.x) and ϕ(.x) − G(.x).

$\Phi(x) = [\Phi(x) - G(x)]_{graph} + [G(x)]_{graph}$

$\Phi_1(x) = 2x^2 [G(x)]_{graph}$

eqs (3) ff.

For $x = 1$: $f = 1$

$\Phi_1(x) = 0.428$

Figure 1. From Ref. 2, Fig. 11-6.
$G(x) = \Phi_1(x)/2x^2$.